SG – 287

26

VI Semester B.A./B.Sc. Examination, Sept./Oct. 2021
(CBCS) (F+R) (2016-17 and Onwards)
MATHEMATICS (Paper – VII)

Time : 3 Hours   Max. Marks : 70

Instruction : *Answer **all** Parts.*

PART – A

Answer **any five** questions : **(5×2=10)**

1. a) Define a vector space over a field.

   b) Show that $w = \{(0, 0, z) \mid Z \in R\}$ is a subspace of V_3 (R) .

   c) For what value of K the set of vectors (3, 2, –1), (0, 4, 5) and (6, K, –2) of $V_3(R)$ is linearly dependent.

   d) Find the matrix of linear transformation $T : V_2(R) \rightarrow V_3(R)$ defined by $T(x, y) = (3x - y, 2x + 4y, 5x - 6y)$ with respect to the standard bases.

   e) Write the scalar factors in cylindrical co-ordinate system.

   f) Solve : $\frac{dx}{zx} = \frac{dy}{yz} = \frac{dz}{xy}$.

   g) Form a partial differential equation by eliminating the arbitrary constants from $z = ax + by + ab$.

   h) Solve $\sqrt{p} + \sqrt{q} = 1$.

PART – B

Answer **two full** questions. **(2×10=20)**

2. a) State and prove the necessary and sufficient condition for a non-empty subset w of a vector space V(F) to be a subsapce of V.

   b) Find the basis and dimension of the subspace spanned by (1, – 2, 3) (1, –3, 4), (–1, 1, –2) of the vector space $V_3(R)$.

OR

P.T.O.

3. a) Show that the intersection of any two subspace of a vector space V(F) is also a subspace of V(F).

   b) Prove that the subset $W = \{(x_1, x_2, x_3)/x_1 + x_2 + x_3 = 0\}$ is a subspace of $V_3(R)$.

4. a) Find the linear transformation $T : R^2 \to R^3$ such that $T(-1, 1) = (-1, 0, 2)$, $T(2, 1) = (1, 2, 1)$.

   b) Find the linear transformation of the matrix $\begin{pmatrix} -1 & 2 & 1 \\ 1 & 0 & 3 \end{pmatrix}$ relative to the bases $B_1 = \{(1, 2, 0), (0, -1, 0), (1, -1, 1)\}$ and $B_2 = \{(1, 0), (2, -1)\}$

OR

5. a) Let $T : V_3(R) \to V_3(R)$ be a linear transformation such that $T(1, 0, 0) = (1, 0, 2)$, $T(0, 1, 0) = (1, 1, 0)$, $T(0, 0, 1) = (1, -1, 0)$. Find the range, null space, rank, nullity and hence verify rank-nullity theorem.

   b) Show that the linear transformation $T : R^3 \to R^3$ given by $T(e_1) = e_1 + e_2$, $T(e_2) = e_1 - e_2 + e_3$, $T(e_3) = 3e_1 + 4e_3$ is non-singular. Where $\{e_1, e_2, e_3\}$ is the standard basis of R^3.

PART – C

Answer **two full** questions : **(2×10=20)**

6. a) Verify the condition for integrability and solve $z^2dx + (z^2 - 2yz)\,dy + (2y^2 - yz - xz)dz = 0$.

   b) Solve $(y - z)p + (z - x)q = x - y$.

OR

7. a) Show that the cylindrical coordinate system is orthogonal curvilinear co-ordinate system.

   b) Express the vector $\vec{f} = z\hat{i} - 2x\hat{j} + y\hat{k}$ in terms of spherical coordinates and find f_r, f_θ, f_φ.

8. a) Solve : $\dfrac{dx}{\tan x} = \dfrac{dy}{\tan y} = \dfrac{dz}{\tan z}$.

   b) Solve : $(mz - ny)\,p + (nx - lz)q = ly - mx$.

OR

9. a) Express the vector $\vec{f} = 2x\hat{i} - 2y^2\hat{j} + xz\hat{k}$ in cylindrical coordinate system and find f_ρ, f_φ, f_z.

   b) Express the vector $\vec{f} = z\hat{i} - 2x\hat{j} + y\hat{k}$ in spherical coordinates system and find f_r, f_θ, f_φ.

PART – D

Answer **two full** questions. **(2×10=20)**

10. a) Form a partial differential equation by eliminating arbitrary function from $z = f(x + ay) + g(x - ay)$.

    b) Solve : $p^2 - q^2 = x - y$.

OR

11. a) Solve $(D^2 - 5DD' + 4D'^2)z = \sin(4x + y)$.

    b) Solve $x^2 p^2 + y^2 q^2 = z^2$.

12. a) Solve : $px + qy = pq$ by Charpit's method.

    b) Solve : $(D^2 - DD' - 6D'^2)z = xy$.

OR

13. a) Solve $\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$ subject to the conditions

    i) $u(0, t) = 0, u(l, t) = 0 \; t \geq 0$.

    ii) $u(x, 0) = \frac{100x}{l} \quad 0 \leq x \leq l$.

    b) Solve $\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}$ given

    i) $u(0, t) = 0, u(l, t) = 0$

    ii) $u(x, 0) = k(lx - x^2)$; $\left(\frac{\partial u}{\partial t}\right)_{t=0} = 0$.