SG – 288

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VI Semester B.A./B.Sc. Examination, September/October 2021
(CBCS) (F + R) (2016-17 and Onwards)
Paper – VIII : MATHEMATICS

Time : 3 Hours | Max. Marks : 70

Instruction : *Answer **all** Parts.*

PART – A

Answer **any five** questions : **(5×2=10)**

1. a) Evaluate : $\lim\limits_{z\to 1+i}\left(z^2+2z\right)$.

   b) Prove that $U=\frac{1}{2}\log\left(x^2+y^2\right)$ is harmonic.

   c) Show that the function defined by $f(z)=\begin{cases}\dfrac{x^3-y^3}{x^3+y^3}, & z\neq 0\\ 0, & z=0\end{cases}$ is not continuous at the origin.

   d) Define cross ratio of four points.

   e) Verify Cauchy-Reimann equations for f(z) = sinx coshy + i cosx sinhy.

   f) State fundamental theorem of Algebra.

   g) Find the real root of the equation $x^3 - x - 2 = 0$ over the interval (1.5, 2) upto two approximation by bisection method.

   h) Using Newton-Raphson method, find the real root of $x^2 + 5x - 11 = 0$ in (1, 2) in one iteration only.

PART – B

Answer **four full** questions : **(4×10=40)**

2. a) Show that the locus of $\arg\left(\frac{\bar{z}}{2}\right)=\frac{\pi}{2}$ is a line through the origin.

   b) State and prove necessary conditions for a function f(x, y) = u(x, y) + iv (x, y) to be analytic.

OR

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3. a) Evaluate $\lim\limits_{z \to 2e^{i\pi/3}} \left(\dfrac{z^3 + 8}{z^4 + 4z^2 + 16} \right)$.

   b) Show that f(z) = sin z is analytic and hence f′(z) = cos z.

4. a) Prove that $u = x^3 - 3xy^2$ is harmonic and find the analytic function whose real part is u(x, y).

   b) If f(z) = u + iv is an analytic function, then prove that the curves $u(x, y) = c_1$, $u(x, y) = c_2$ form two orthogonal families.

OR

5. a) If f(z) = u + iv is analytic, then show that

$$\left[\frac{\partial}{\partial x} | f(z) |\right]^2 + \left[\frac{\partial}{\partial y} | f(z) |\right]^2 = | f'(z) |^2.$$

   b) If f(z) = u + iv and $u - v = e^x(\cos y - \sin y)$, find f(z) interms of z.

6. a) Evaluate $\int\limits_{(0,1)}^{(2,5)} (3x + y)\,dx + (2y - x)\,dy$ along the curve $y = x^2 + 1$.

   b) State and prove Cauchy's integral formula.

OR

7. a) Evaluate $\int\limits_C \dfrac{dz}{z^2 - 4}$ over C : |z| = 1.

   b) State and prove Cauchy's integral theorem.

8. a) Show that the transformation $w = \dfrac{1}{z}$ transforms a circle to circle or to a straight line.

   b) Discuss the transformation of $w = z^2$.

OR

9. a) Find the Bilinear transformation which map the points z = 1, i, – 1 into w = i, 0, – i.

   b) Show that the transformation $w = \dfrac{i - z}{i + z}$ maps the x – axis of the z-plane onto a circle |w| = 1 and points in the half plane y > 0 on the points |w| < 1.

PART – C

Answer **two full** questions : **(2×10=20)**

10. a) Find the root of the equation $x^3 - 4x + 1 = 0$ by Regula-Falsi method upto three decimal places.

 b) Using Newton-Raphson method, find the real root of $x^3 + 5x - 11 = 0$ near 1 correct to 3 decimal places.

OR

11. a) Solve the equations

$x + y + 54z = 110$

$27x + 6y - z = 85$

$6x + 15y + 2z = 72$ by Gauss-Seidal Method.

 b) Find the largest eigenvalue of the matrix

$$\begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$$ by power method.

12. a) Use Taylor's series method to find y at x = 0.1 considering terms upto the third degree given $\frac{dy}{dx} = x^2 + y^2$ and y(0) = 1.

 b) Using modified Euler's method, find y(0.1), given $\frac{dy}{dx} = x^2 + 1$, y(0) = 1.

OR

13. a) Given $\frac{dy}{dx} = \frac{y-x}{y+x}$, with y(0) = 1, find y, approximately for x = 0.1 by Euler's method in five steps.

 b) By using Runge-Kutta method, solve $\frac{dy}{dx} = 3x + \frac{y}{2}$ with y(0) = 1. Compute y(0.2) by taking h = 0.2.