UG – 093

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V Semester B.A./B.Sc. Examination, March/April 2021
(Semester Scheme)
(CBCS) (F + R) (2016 – 17 and Onwards)
MATHEMATICS – V

Time : 3 Hours Max. Marks : 70

Instruction : *Answer **all** questions.*

PART – A

Answer **any five** questions. **(5×2=10)**

1. a) In a ring (R, +, •), prove that $a \cdot (b - c) = a \cdot b - a \cdot c \; \forall \; a, b, c \in R$.

   b) Define field. Give an example.

   c) If f is a homomorphism of a ring R into ring R′ then prove that f(0) = 0′ where 0 is the zero element of R and 0′ is the zero element of R′.

   d) Find the divergence of the vector field $\vec{F} = x^3z\,\hat{i} + y^3x\,\hat{j} + z^3y\,\hat{k}$ at (1, 1, –1).

   e) Find the maximum directional derivative of $\phi = x^3y^2z$ at (1, –2, 3).

   f) Evaluate $\Delta^3(1 - ax)(1 - bx)(1 + cx)$.

   g) Write the Newton backward interpolation formula.

   h) State Simpson's $\frac{3}{8}^{th}$ rule for the integral $\int_a^b f(x)dx$.

PART – B

Answer **two full** questions. **(2×10=20)**

2. a) Prove that the set of all matrices of the form $M = \left\{ \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix} \middle/ a, b \in Q \right\}$ is a non-commutative ring without unity w.r.t. addition and multiplication of matrices.

   b) Prove that $(z_6, +_6, \times_6)$ is a ring w.r.t. $+_6$ and $\times_6$.

OR

P.T.O.

3. a) Prove that a ring R is without zero divisors if and only if the cancellation laws holds in R.
   b) Prove that every field is an integral domain.

4. a) Prove that $(z_5, +_5, \times_5)$ is a commutative ring with unity. Is it an integral domain ?
   b) Prove that the set $S = \left\{ \begin{bmatrix} a & 0 \\ b & 0 \end{bmatrix} \Big/ a, b \in z \right\}$ of all 2×2 matrices is a left ideal of the ring R over z. Also show that S is not a right ideal.

OR

5. a) State and prove fundamental theorem of homomorphism.
   b) If I is an ideal of the ring R, then prove that the quotient ring R/I is homomorphic image of R with I as its Kernel.

PART – C

Answer **two full** questions. **(2×10=20)**

6. a) Prove that the surfaces $4x^2y + z^3 = 4$ and $5x^2 - 2yz = 9$ intersect orthogonally at the point (1, –1, 2).
   b) Find the directional derivative of $\phi(x, y, z) = x^2yz + 4z^2$ at the point (1, –2, –1) in the direction of $2\hat{i} - \hat{j} - 2\hat{k}$.

OR

7. a) Show that $\vec{F} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational. Find ϕ such that $\vec{F} = \nabla\phi$.
   b) If $\vec{F} = (3x^2y - z)\hat{i} + (xz^3 + y)\hat{j} - 2x^3z^2\hat{k}$. Find curl $\left(\text{curl } \vec{F}\right)$.

8. a) If $\vec{f} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$ and $\vec{g} = yz\hat{i} + zx\hat{j} + xy\hat{k}$, then prove that $\vec{f} \times \vec{g}$ is solenoidal.
   b) If ϕ is scalar point function and $\vec{F}$ is vector point function then prove that $\text{curl}\left(\phi\vec{F}\right) = \phi \text{ curl } \vec{F} + (\text{grad } \phi) \times \vec{F}$.

OR

9. a) Prove that (i) $\text{div}\left(\text{curl } \vec{F}\right) = 0$
   (ii) $\text{curl}\,(\text{grad } \phi) = 0$
   b) Prove that $\text{div}\left(\vec{f} \times \vec{g}\right) = \vec{g} \cdot \left(\text{curl } \vec{f}\right) - \vec{f} \cdot \left(\text{curl } \vec{g}\right)$.

PART – D

Answer **two full** questions. **(2×10=20)**

10. a) Find a second degree polynomial which takes the following data.

x	1	2	3	4
f(x)	–1	–1	1	5

b) Find f(84) from the following data.

x	40	50	60	70	80	90
y = f(x)	184	204	226	250	276	304

OR

11. a) Use the method of separation of symbols to prove that

$$U_0 + \frac{U_1 x}{1!} + \frac{U_2 x^2}{2!} + \frac{U_3 x^3}{3!} + \ldots.\infty = e^x \left[U_0 + \frac{x\Delta U_0}{1!} + \frac{x^2 \Delta^2 U_0}{2!} + \ldots.\infty \right].$$

b) Obtain the function whose first difference is $9x^2 + 11x + 5$.

12. a) Using Newton's divided difference formula. Find f(10) from the following data.

x	4	7	9	12
f(x)	–43	84	327	1053

b) Evaluate $\int_0^{\pi/2} \sqrt{\cos\theta} \cdot d\theta$ by using Simpson's $\frac{1}{3}^{rd}$ rule dividing $\left[0, \frac{\pi}{2}\right]$ into six equal parts.

OR

13. a) Using Lagrange's interpolation formula. Find f(2) from the following data.

x	0	1	3	4
f(x)	5	6	50	105

b) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Trapezoidal rule. Divide [0, 6] into six sub intervals.