

# ED-2814

## M. A./M. Sc. (Final) EXAMINATION, 2021

MATHEMATICS

(Optional)

Paper Fifth (iii)

(Fuzzy Sets and Their Application)

*Time : Three Hours*

*Maximum Marks : 100*

**Note :** Attempt any *two* parts from each question. All questions carry equal marks.

1. (a) If :

$$A = \frac{1}{1.0} + \frac{2}{0.8} + \frac{3}{0.5} + \frac{4}{0.1}$$

is defined on  $X = 1, 2, 3, 4$ . Find all its  $\alpha$ -cuts sets. Also prove that  $A$  can be expressed in terms of the family composed of all of its  $\alpha$ -cuts.

(b) Define  $t$ -norm with example and prove that the standard fuzzy intersection is the only idempotent  $t$ -norm.

- (c) If A is a fuzzy set defined on X, then prove that  $\bigcup_{\alpha \in [0, 1]} \alpha_+ A = A_1$  where  $\alpha_+ A$  is a special set and

$$\alpha_+ A(x) = \alpha \cdot \alpha_+ A(x).$$

2. (a) If R is fuzzy relation on  $X^2$ . Then prove that fuzzy relation :

$$R_{T(i)} = \bigcup_{n=1}^{\infty} R^{(n)}$$

is the smallest  $i$ -transitive closure of R.

- (b) Define symmetric fuzzy relation with example. If R is symmetric fuzzy relation, then prove that each power of R is symmetric.
- (c) What do you mean by sagittal diagram. Draw the sagittal diagram for the following fuzzy relation R :

$$R = \begin{matrix} & \begin{matrix} y_1 & y_2 & y_3 & y_4 \end{matrix} \\ \begin{matrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{matrix} & \begin{bmatrix} .8 & 0 & .3 & .4 \\ .3 & 1 & .8 & 0 \\ .7 & 0 & 1 & 1 \\ .4 & .5 & 0 & 0 \\ 0 & 1 & .5 & .8 \end{bmatrix} \end{matrix}$$

3. (a) Solve the following fuzzy relation equation using max-min operations :

$$P \circ Q \begin{bmatrix} .9 & .6 & 1 \\ .8 & .8 & .5 \\ .6 & .4 & .6 \end{bmatrix} = [.6 \ .6 \ .5]$$

- (b) If  $m_1$  and  $m_2$  are basic probability assignments on  $X = a, b, c, d$  which are obtained from two independent sources be defined as follows :

$$m_1 \quad a, b = .2$$

$$m_1 \quad b, d = .5$$

$$m_1 \quad a, c = .3$$

$$m_2 \quad a, d = .2$$

$$m_2 \quad b, c = .5$$

$$m_2 \quad a, b, c = .3$$

Calculate the combined basic probability assignment  $m_{1.2}$  by using the Dempster rule of combination.

- (c) Prove that :

(i)  $\text{bel}(A) + \text{bel}(\bar{A}) \leq 1$

(ii)  $\text{pl}(A) + \text{pl}(\bar{A}) \geq 1$

4. (a) What do you mean by interpolation method ? If  $A_1, A_2, B_1, B_2$  are fuzzy sets such that :

$$A_1 = \frac{1}{x_1} + \frac{.9}{x_2} + \frac{.1}{x_3}$$

$$A_2 = \frac{.9}{x_1} + \frac{1}{x_2} + \frac{.2}{x_3}$$

$$B_1 = \frac{1}{y_1} + \frac{.2}{y_2}$$

$$B_2 = \frac{.2}{y_1} + \frac{.9}{y_2}$$

X is A is fact where  $A = \frac{.8}{x_1} + \frac{.9}{x_2} + \frac{.1}{x_3}$

Then calculate the conclusion B by method of interpolation.

- (b) Write short notes on the following :

- (i) unconditional and unqualified proposition

- (ii) relative quantifier
  - (iii) multivalued logic
  - (iv) linguistic hedges
- (c) If  $X = \{1, 2, 3, 4\}$  and  $Y = \{1, 2, 3, 4, 5, 6\}$  are two universe of discourse and

$$A = \frac{0.6}{2} + \frac{1}{3} + \frac{0.2}{4}$$

$$B = \frac{0.4}{2} + \frac{1}{3} + \frac{0.8}{4} + \frac{0.3}{5}$$

Apply the fuzzy modus ponens rule to find the relation. If  $x$  is A, then  $y$  is B.

5. (a) If each individual of four decisions makes has a total preference ordering  $P_i (i \in N)$  on a set of alternatives  $X = \{a, b, c, d\}$  as :

$$P_1 = \{a, b, d, c\}$$

$$P_2 = \{a, c, b, d\}$$

$$P_3 = \{b, a, c, d\}$$

$$P_4 = \{a, d, b, c\}$$

Find the fuzzy preference relation. Also find the  $\alpha$ -cuts of the fuzzy relation.

- (b) Solve the following fuzzy linear programming problem :

Maximize :

$$z = 5x_1 + 4x_2$$

$$(4, 2, 1)x_1 + (5, 3, 1)x_2 \leq (24, 5, 8)$$

$$(4, 1, 2)x_1 + (1, .5, 1)x_2 \leq (12, 6, 3)$$

$$x_1, x_2 \geq 0$$

- (c) Discuss the methods of defuzzification.