ED-2807

M.A./M.Sc. (Final) Examination, 2021

MATHEMATICS

Compulsory

Paper - II

Partial Differential Equations and Mechanics

Time : Three Hours] [*Maximum Marks* : 100

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) (*i*) Derive Non Homogeneous problem for transport equation.

(*ii*) State and prove the mean value formula for Laplace's equation.

(*b*) Derive fundamental solution for Heat equation.

(*c*) Obtain solution for $n = 3$ of wave equation by spherical means.

Unit-II

2. (*a*) (*i*) State and prove the Hopf Lax formula.

(*ii*) The function $X(\cdot)$ and $P(\cdot)$ satisfy Hamilton's equation

$$\dot{X}(s) = D_P H\left(P(s), X(s)\right)$$

$$\dot{P}(s) = -D_x H\left(P(s), X(s)\right)$$

for $0 \le s \le t$, furthermore the mapping $S \to H\left(P(s), X(s)\right)$ is constant.

(*b*) Derive Barenblatt solution to the porous medium equation.

(*c*) State and prove the Cauchy-Kovalevskaya theorem.

Unit-III

3. (*a*) Derive equation of motion in generalized co-ordinates for Holonomic dynamical system.

(*b*) Derive Euler-Poisson equation.

(*c*) Derive Routh's equation of motion.

Unit-IV

4. (*a*) Derive principle of Least action.

(*b*) The transformation equations between two sets of co-ordinates are

$$Q = \log\left(1 + \sqrt{q}\cos p\right),$$

$$P = 2\left(1 + \sqrt{q}\cos p\right)\sqrt{q}\sin p$$

show that

(*i*) These transformations are canonical if q, p are canonical.

(*ii*) The generating function

$$F_3 = -\left(C^{\theta} - 1\right)^2 \tan p.$$

(*c*) Derive invariance of Lagrange's bracket's under canonical transformation.

Unit-V

5. (*a*) To find the attraction of a thin uniform spherical shell of an external, internal and surface point P.

(*b*) Derive Poisson's equation for spherical polar co-ordinates.

(*c*) (*i*) Derive relation between the potential and attraction.

(*ii*) The density of an elliptic Lamina varies as the distance from the major axis, the mass at a unit element of area at a unit distance being μ. Show that the potential due to the Lamina at the focus is $2Y\mu b^2$.

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