

ED-2802

M.A./M.Sc. (Previous) Examination, 2021

MATHEMATICS

Paper - II

Real Analysis

Time: Three Hours] [Maximum Marks: 100

Note: Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

- 1. (a) (i) Define unit step function.
 - (ii) Suppose $C_n \ge 0$ for $n = 1, 2, 3, \sum C_n$ converges, $\{s_n\}$ is a sequence of distinct points in (a, b) and

$$\alpha(a) = \sum_{n=1}^{\infty} C_n I(x - s_n).$$

Let f be continuous on [a, b]; then

prove that
$$\int_{a}^{b} f d_{\alpha} = \sum_{n=1}^{\infty} C_{n} f(s_{n}).$$

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(Turn Over)

(2)

- (b) Let I = [0, 1] and let $f, \alpha: I \to R$ be function such that $f(x) = \alpha(x) = x^2$. Then find the value of $\int_0^1 x^2 dx$.
- (c) Let $Y:[a,b] \to \mathbb{R}^k$ be a curve. If $C \in (a,b)$, then prove that $\Lambda_Y(a,b) = \Lambda_Y(a,c) + \Lambda_Y(c,b)$.

 Unit-II

- (a) State and prove the Riemann's theorem 2. on rearrangement of series.
 - (b) State and prove the Abel's test for uniform convergence.
 - (c) State and prove the converse of Abel's theorem.

Unit-III

- (a) State and prove the Chain Rule.
 - (b) Show that the volume of the greatest rectangular parallelepiped inscribed in the

ellipsoid
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$
 is $\frac{\delta abc}{3\sqrt{3}}$.

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(Continued)

(3)

(c) Let ψ be a k-chain of class $\boldsymbol{\mathcal{C}}$ in an open set $V \subset R^m$ and let ω be a (k-1)-form of class $\boldsymbol{\mathcal{C}}$ in V. The prove that $\int_{\psi} d\omega = \int_{\partial \psi} \omega.$

Unit-IV

- **4.** (a) Prove that the outer measure of an interval is its length.
 - (b) State and prove the lebesgue's dominated convergence theorem.
 - (c) State and prove the fundamental theorem of integral calculus.

Unit-V

- 5. (a) Prove that the set function μ^* is an outer measure.
 - (b) State and prove the Riesz-Fischer theorem.
 - (c) State and prove the Riesz theorem.

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