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ED-2803

M.A./M.Sc. (Previous) EXAMINATION, 2021

MATHEMATICS

Paper Third

(Topology)

Time : Three hours **Maximum Marks : 100**

Note : *All questions are compulsory. Solve any two parts of each question. All questions carry equal marks.*

Unit–1

1. (a) State and prove Schroeder-Bernstein theorem.

(b) Let (X, T) be a topological space and $A \quad X$. Then int (A) is the union of all open sets contained in A. It is also the largest open subsets of X contained in A.

(c) Define relative topology. Let (X, T) be a topological space and $Y \quad X$. Let $V \quad T / Y$ such that $V \quad U \quad Y$ where $U \quad T$. Show that T / Y is topology on Y and hence $(Y, T / Y)$ is topological space.

Unit–2

2. (a) Define homeomorphism. Prove that homeomorphism is an equivalence relation in the family of topological spaces.

(b) State and prove Lindelof's theorem.

(c) Prove that completely regularity is a hereditary property.

Unit–3

3. (a) Prove that a subset of R is compact if and only if it is closed and bounded.

(b) Show that a subset of R is connected if and only if it is an interval.

(c) Define component. Show that in a topological space (X, T), each element of X is contained in exactly one component of X.

Unit–4

4. (a) Show that the product space $X \quad \{X_i : i \quad I\}$ is regular if each coordinate space X_i is regular.

(b) Show that a compact Hausdorff space is separable and metrizable if it is second countable.

(c) Define paracompact space. Show that every metrizable space is paracompact.

Unit–5

5. (a) A topological space (X, T) is Hausdorff if and only if every net in X can converge to atmost one point.

(b) Define filter and ultrafilter. Show that every filter is contained in an ultrafilter.

(c) Define fundamental group of circle. Let $x_0 x_1$ X. If there is a path in X from x_0 to x_1, then the groups $_1(X_1 x_0)$ and $_1(X_1 x_1)$ are isomorphic.