DD-2852

BCA (Part-I) Examination, 2021

Paper - II

Calculus and Statistical Methods

Time : Three Hours] [*Maximum Marks* : 50

Note : Answer any **two** parts from each question. All questions carry equal marks. Only simple calculator is allowed.

Unit-I

1. (*a*) Find the value of

$$\lim_{x \to 0} \frac{(1+x)^{1/x} - e}{x}$$

(*b*) Test the following function for continuity at $x = 0$:

$$f(x) = \begin{cases} \dfrac{3x + 4\tan x}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$$

(*c*) Let $f(x) = x\dfrac{e^{\frac{1}{x}} - e^{\frac{-1}{x}}}{e^{\frac{1}{x}} + e^{\frac{-1}{x}}}$ for $x \neq 0, f(0) = 0$.

Show that f is continuous but not differentiable at $x = 0$.

Unit-II

2. (*a*) Differentiate the following function w.r.t.x

$$\frac{\sec x + \tan x}{\sec x - \tan x}$$

(*b*) If $x = a \sin 2\theta\,(1 + \cos 2\theta)$ and $y = a \cos 2\theta\,(1 - \cos 2\theta)$, show that $\dfrac{dy}{dx} = \tan\theta$.

(*c*) If $y^{\cot x} + (\tan^{-1}x)^{y} = 1$, find $\dfrac{dy}{dx}$.

Unit-III

3. (*a*) Prove that the curve $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 1$ touches the straight line $\frac{x}{a} + \frac{y}{b} = 2$ at the point (a, b), whatever the value of n.

(*b*) Explain Monotonic Increasing and Decreasing functions.

(*c*) Investigate for what value of x, $5x^6 - 18x^5 + 15x^4 - 10$ is a maximum or minimum.

Unit-IV

4. (*a*) The chance of One event happening is the square of the chance of a Second event happening, but the odds against the first are the cube of the odds against the second. Find the chance of happening of each.

(*b*) If E_1 and E_2 are any two events then,

$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

(*c*) A card is drawn from an ordinary pack of cards and a player bets that it is a spade or an ace. What are the odds against his winning the bet ?

Unit-V

5. (*a*) Show that in a discrete series, if the deviations are small compared with the mean M so that $\left(\frac{X}{M}\right)$ and higher powers of X/M can be neglected. Prove that approximately :

(*i*) $G = M\left[1 - \frac{6^2}{2M^2}\right]$

(*ii*) $M^2 - G^2 = 6^2$

Where the letters have their usual meaning.

(*b*) Fit Poisson's distribution to the following and calculate theoretical frequencies ($e^{-0.5} = 0.61$) :

Deaths	:	0	1	2	3	4
Frequency	:	122	60	15	2	1

(*c*) The two regression equations of the variables x and y are $x = 19.13 - 0.87\,y$ and $y = 11.64 - 0.50\,x$. Find

(*i*) mean of x's

(*ii*) mean of y's and

(*iii*) the correlation coefficient between x and y.

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