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# ED-2862

## B.C.A. (Part-II) EXAMINATION, 2021

### CALCULUS AND DIFFERENTIAL EQUATIONS

#### Paper First

Time : Three hours

Maximum Marks : 80

Note : All questions are compulsory. Attempt any two parts from each Unit. All questions carry equal marks. Simple / Scientific calculator is allowed.

#### Unit-1

1. (a) Test the continuity of the function at the origin.

$$f(x) = \begin{cases} \frac{1}{xe^x} & \text{if } x > 0 \\ 1 - e^x & \text{if } x < 0 \end{cases}$$

(b) Prove that if the function  $f$  is continuous in the closed interval  $[a, b]$  then  $f$  is bounded in this interval.

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(c) Test the differentiability of the function

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x} & \text{when } x \neq 0 \\ 0 & \text{when } x = 0 \end{cases}$$

at  $x = 0$ .

#### Unit-2

2. (a) If  $y^{\cot x} = (\tan^{-1} x)^y - 1$ . Find  $\frac{dy}{dx}$ .

(b) If  $x^y = e^{x-y}$ , prove that :

$$\frac{dy}{dx} = \frac{\log x}{(1 - \log x)^2}$$

(c) Find the maximum and minimum value of  $x^{5/3} - 5x^{2/3}$ .

#### Unit-3

3. (a) Evaluate :

$$\int \frac{dx}{(2x-1)\sqrt{4x-3}}$$

(b) Evaluate :

$$\int \frac{dx}{3 \cos x - 4 \sin x - 5}$$

(c) Evaluate :

$$\int \frac{xe^x}{(1-x)^2} dx$$

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**Unit-4**

4. (a) Find the value of :

$$\int_0^1 \frac{x \tan^{-1} x}{(1-x^2)^{\frac{3}{2}}} dx$$

(b) Prove that :

$$\int_0^{\pi/2} \log \sin x \, dx = -\frac{\pi}{2} \log 2$$

(c) Prove that :

$$\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} \sqrt{\cos x}} dx = \frac{\pi}{4}$$

**Unit-5**

5. (a) Form the differential equation

$$y = A \cos(x^2) + B \sin(x^2)$$

(b) Solve the differential equation :

$$(x^2 - yx^2)dy - (y^2 - xy^2)dx = 0$$

(c) Solve the equation :

$$\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}} = 0$$