Roll No. ....

DD–2863

B. C. A. (Part II) EXAMINATION, 2021

Paper Second

DIFFERENTIATION AND INTEGRATION

Time : Three Hours

Maximum Marks : 50

Note : All questions are compulsory. Attempt any *two* parts from each question. All questions carry equal marks.

Unit—I

1. (a) If :

$$\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$$

then prove that :

$$x^2 y_{n+2} + (2n+1)x\,y_{n+1} - (n^2 - m^2)\,y_n = 0$$

(b) By Tylor's theorem prove that :

$$\log(x+h) = \log h + \frac{x}{h} - \frac{x^2}{2h^2} + \frac{x^3}{3h^3} - ....$$

P. T. O.

(c) Use Tylor's theorem prove that :

$$\tan^{1}(x+h)=\tan^{-1}x+h\sin z.\frac{\sin z}{1}-(h\sin z)^{2}\frac{\sin 2z}{2}+$$

$$(h\sin z)^{3}.\frac{\sin 3z}{3}+.......+(-1)^{n-1}(h\sin z)^{2}.\frac{\sin n^{2}}{n}$$

$$+.....z=\cot^{-1}x$$

Unit—II

2. (a) Find the asymptotes of the curve,

$$x^{3}+2x^{2}y-xy^{2}-2y^{3}+4y^{2}+2xy+y-1=0$$

(b) Find the Pedal formula for Radius of curvature and also define Pedal equation.

(c) Trace the curve $ay^{2}=x^{3}$.

Unit—III

3. (a) If

$$x^{x}y^{y}z^{z}=c$$

then show that :

$$\frac{\partial^{2}z}{\partial x\partial y}=-(x\log ex)^{-1}$$

when $x=y=z$.

(b) Find the directional derivative of

$$f(x,y,z)=x^{2}+xy+z^{2}$$

at the point $A(1,-1,1)$ in the direction of the line AB, where B has coordinate (3, 2, 1).

(c) If

$$x + y + z = u$$
$$y + z = uv$$
$$z = uvw$$

then prove that :

$$\frac{\partial(x\ y\ z)}{\partial(u\ v\ w)} = u^2 v$$

Unit—IV

4. (a) Evaluate :

$$\int \frac{x^2}{x^4 + x^2 + 1}\, dx$$

(b) Evaluate :

$$\int_0^{\pi/2} \frac{x}{\sin x + \cos x}\, dx$$

(c) Evaluate :

$$\int \frac{x + \sin x}{1 + \cos x}\, dx$$

Unit—V

5. (a) Evaluate :

$$\int_0^{\log 2} \int_0^{x} \int_0^{x+\log y} e^{x+y+z}\, dx\, dy\, dz$$

(b) Change the order of an integration :

$$\int_0^{a} \int_0^{x} f(xy)\, dx\, dy$$

(c) Prove that the arc length of the curve $y = \log \sec x$ from $x = 0$ to $x = \frac{\pi}{3}$ is $\log_e(2 + \sqrt{3})$.