Roll No. ....

DD-2871 (SE)

B. C. A. (Part III) EXAMINATION, 2020

Paper First

CALCULUS AND GEOMETRY

Time : Three Hours

Maximum Marks : 50

Note : All questions are compulsory. Attempt any *two* parts from each Unit. All questions carry equal marks.

Unit—I

1. (a) Show that the constant function k is integrable and $\int_a^b k\,dx = k(b-a)$.

(b) A necessary and sufficient condition for integrability of a bounded function f is that to every $\in > 0$ there corresponds $\delta > 0$ such that for every partition P of $[a,b]$ with norm $\mu(P) < \delta$

$$U(P,f) - L(P,f) < \in$$

(c) If a function f is a continuous on $[a,b]$, then there exists a number α in $[a,b]$ such that :

$$\int_a^b f(x)\,dx = f(\alpha)(b-a)$$

f is a continuous, therefore $f \in R[a,b]$.

Unit—II

2. (a) Discuss the maximum and minimum values of the function $f(x, y) = x^3 - 4xy + 2y^2$.

(b) Find three positive numbers whose sum is 30 and whose product is maximum.

(c) Prove that of all rectangular parallelepipeds of the same volume, the cube S has the least surface.

Unit—III

3. (a) Show that :

$$\int_0^1 x^{m-1}(1-x)^{n-1}dx$$

exists if and only if m, n both are positive.

(b) Examine the covergence of the integral $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)}$.

(c) Test the convergence for the function :

$$\int_a^{\infty} e^{-x} \frac{\sin x}{x^2} dx, \text{ where } a > 0.$$

Unit—IV

4. (a) Find the equation of a cone whose vertex is at origin and direction cosine of its generators satisfying the relation $4l^2 + 7m^2 - 8n^2 = 0$.

(b) Find the angle between the lines in which the plane $2x + y - z = 0$ cuts the cone $4x^2 - y^2 + 3z^2 = 0$. Find also the angle between the lines of section.

(c) Find the equation of the cylinder whose generators are parallel to the line $x = \frac{-y}{2} = \frac{z}{3}$ and whose guiding curve is the ellipse $x^2 + y^2 = 1, z = 3$.

Unit—V

5. (a) **Explain the relation between Cartesian and Polar co-ordinantes.**

(b) In an ellipse $\frac{2}{r} = 1 - e\cos\theta$ if PQ is a chord passing through focus S, then prove that :

$$\frac{1}{SP} + \frac{1}{SQ} = 1$$

(c) To find the Polar equation of a conic with its latus rectum of length $2l$, eccentricity e and the focus being pole.

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