Roll No. .....

DD-2872 (SE)

B. C. A. (Part III) EXAMINATION, 2020

Paper Second

DIFFERENTIAL EQUATION AND FOURIER SERIES

Time : Three Hours

Maximum Marks : 50

Note : All questions are compulsory. Attempt any *two* parts from each question. All questions carry equal marks. Only simple calculator is allowed not scientific calculator.

1. (a) Solve :

$$\frac{dy}{dx} = \frac{4y}{x(y-3)}$$

(b) Solve :

$$y = 2px + p^4x^2$$

(c) Solve :

$$\left(y^2 e^{xy^2} + 4x^3\right)dx + \left(2xy\, e^{xy^2} - 3y^2\right)dy = 0$$

2. (a) Solve :

$$\left(D^2 + 5D + 4\right)y = 3 - 2x$$

(b) Solve :

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 4y = \sin 2x$$

(c) Solve :

$$x^3\frac{d^3y}{dx^3} + 3x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$$

3. (a) Solve :

$$(z-y)p + (x-z)q = (y-x)$$

(b) Solve :

$$p^2 - q^2 = 1$$

(c) Solve :

$$(D^2 - 2D')^2 (D + 3D')z = e^{2x+y}$$

4. (a) Find the Fourier series of the function $f(x) = x + x^2$ in the interval $(0, 2\pi)$.

(b) Obtain the Fourier expansion of the function e^{ax} in the interval $-\pi \le x \le \pi$.

(c) Obtain the Fourier sine series of the function $f(x) = x$ in the interval $(0, \pi)$.

5. (a) What is the piecewise continuous function ? Explain.

(b) Discuss the convergence of the full Fourier series.

(c) The piecewise smooth function defined by the equations :

$$f(x) = \begin{cases} -\pi / 2 & \text{when} \quad -\pi < x < 0 \\ \pi / 2 & \text{when} \quad 0 < x < \pi \end{cases}$$

and $f(0) = 0$. Prove that the Fourier sine series is :

$$2\sum_{n=1}^{\infty} \frac{\sin(2n-1)x}{2n-1}$$

for f converges to $f(x)$ everywhere in the interval $-\pi < x < \pi$.

1650