ED-612

M.A./M.Sc. 3rd Semester
Examination, March-April 2021

MATHEMATICS

Paper - I

Integration Theory and Functional Analysis

Time : Three Hours] [*Maximum Marks* : 80
[*Minimum Pass Marks* : 16

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) Let E be a measurable set such that $0 < \nu E < \infty$. Then there is a positive set A contained in E with $\nu A > 0$.

(*b*) State and prove Radon-Nikodym theorem.

(*c*) State and prove Riesz Representation theorem.

Unit-II

2. (*a*) State and prove Fubini theorem.

(*b*) If f is absolutely continuous on $[a, b]$ and $f'(x) = 0$ a.e. then f is constant.

(*c*) Let E be a measurable subset of $X \times Y$ such that $\mu \times \nu\,(E)$ is finite. Then for almost all x the set E_x is a measurable subset of Y. The function g defined by $g(x) = \nu\,(E_x)$ is a measurable function defined for almost all x and

$$\int g\,d\mu = \mu \times \nu(E)$$

Unit-III

3. (*a*) Let K be a compact set, O an open set with $K \subset O$. Then

$$K \subset U \subset H \subset O$$

where U is a r-compact open set and H is a compact G_δ.

(*b*) Let μ be a Baire measure on a locally compact space X and E a r-bounded Baire set in X. Then for $\epsilon > 0$,

(*i*) There is a r-compact open set O with $E \subset O$ and $\mu\,(O \sim E) < \epsilon$.

(*i*) $\mu\,E = \sup\,\{\mu\,k : K \subset E,\ K$ a compact $G_\delta\}$.

(*c*) State and prove Riesz-Markov theorem.

Unit-IV

4. (*a*) Let M be a closed linear subspace of a normed linear space X. Then the quotient space X/M with the norm

$$\| x + M \| = \inf \{\| x + M \| : m \in M\}$$

is a Banach space if X is a Banach space.

(*b*) Let X be a finite dimensional normed linear space. Then any two norms defined on X are equivalent.

(*c*) Show that a bounded linear transformation T from a normed linear space X into Y is continuous.

Unit-V

5. (*a*) Define weak convergence. Let $\{x_n\}$ be a weakly convergent sequence in a normed space X. Then

(*i*) The weak limit of $\{x_n\}$ is unique,

(*i*) The sequence $\{\| x_n \|\}$ is bounded.

(*b*) The dual of l_1 is isometrically isomorphic to l_∞.

(*c*) If X is a finite dimensional normed linear space, then weakly convergent sequence on it is strong convergent.

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