# ED-618

M.A./M.Sc. 3rd Semester
Examination, March-April 2021

## MATHEMATICS

Optional - A

Paper - IV

Operations Research - I

*Time* : Three Hours] [*Maximum Marks* : 80
[*Minimum Pass Marks* : 16

**Note** : Answer any **two** parts from each question. All questions carry equal marks.

### Unit-I

**1.** (*a*) Use Simplex method to solve the following linear programming problem :

Maximize $z = 6x_1 + 8x_2$

Subject to : $5x_1 + 10x_2 \leq 60$

$4x_1 + 4x_2 \leq 40$

$x_1, x_2 \geq 0$

(*b*) Solve the following linear programming problem using the result of its dual :

Minimize $z = 24x_1 + 30x_2$

Subject to : $2x_1 + 3x_2 \geq 10$

$4x_1 + 9x_2 \geq 15$

$6x_1 + 6x_2 \geq 20$

$x_1, \ x_2 \geq 0$

(*c*) Consider the following linear programming problem :

Maxmize $z = 10x_1 + 15x_2 + 20x_3$

Subject to : $2x_1 + 4x_2 + 6x_3 \leq 24$

$3x_1 + 9x_2 + 6x_3 \leq 30$

$x_1, x_2, x_3 \geq 0$

and check whether the optimality is affected, if the profit coefficients are changed from (10, 15, 20) to (7, 14, 15). If so, find the revised optimum solution.

**Unit-II**

**2.** (*a*) Solve the following linear programming problem using big-M method :

Minimize $z = 2x_1 + 3x_2$

Subject to : $x_1 + x_2 \geq 6$

$7x_1 + x_2 \geq 14$

$x_1, \; x_2 \geq 0$

(*b*) Solve the following linear programming problem using dual simplex method :

Minimize $z = 2x_1 + 4x_2$

Subject to : $2x_1 + x_2 \geq 4$

$x_1 + 2x_2 \geq 3$

$2x_1 + 2x_2 \leq 12$

$x_1, x_2 \geq 0$

(*c*) Find the Dual of the Primal :

Maximize $z = x_1 + 5x_2 + 3x_2$

Subject to : $x_1 + 2x_2 + x_3 = 3$

$2x_1 - x_2 = 4$

$x_1, x_2, x_3 \geq 0$

**Unit-III**

**3.** (*a*) Consider the following parametric linear programming problem :

Maximize $z = (10 - 2t)\, x_1 + (5 - 3t)\, x_2$

Subject to : $8x_1 + 2x_2 \leq 48$

$2x_1 + 4x_2 \leq 24$

$x_1, x_2 \geq 0$

and $t$ is a non-negative parameter. Perform parametric analysis with respect to the objective function coefficients and identify the ranges of $t$ over which the optimality is unaffected.

(*b*) Write a short note on interior point algorithm.

(*c*) Carry out two iterations of Karmarkar's algorithm for the following problem :

Minimize $z = x_1 - 2x_2$

Subject to : $x_1 - 2x_2 + x_3 = 0$

$x_1 + x_2 + x_3 = 1$

$x_1, x_2, x_3 \geq 0$

## Unit-IV

**4.** (*a*) Discuss the similarities and dissimilarities between Transportation and Assignment problem.

(*b*) Use Vogel's approximation method to solve the following transportation problem :

| | Destination 1 | 2 | 3 | 4 | Supply |
|---|---|---|---|---|---|
| Source 1 | 3 | 1 | 7 | 4 | 300 |
| Source 2 | 2 | 6 | 5 | 9 | 400 |
| Source 3 | 8 | 3 | 3 | 2 | 500 |
| Demand | 250 | 350 | 400 | 200 | |

(*c*) Write steps of Hungerian method for solving Assignment problem.

## Unit-V

**5.** (*a*) Write steps of PRIM algorithm for finding the Minimum Spanning Tree problem.

(*b*) A project is composed of 7 activities whose time estimates are listed in the table below. Activities are identified by their beginning ($i$) and ending ($j$) node numbers :

| Activity ($i$-$j$) | Estimate Duration in Weeks | | |
|---|---|---|---|
| | Optimistic ($t_o$) | Most likely ($t_m$) | Pessimistic ($t_p$) |
| 1-2 | 1 | 1 | 7 |
| 1-3 | 1 | 4 | 7 |
| 1-4 | 2 | 2 | 8 |
| 2-5 | 1 | 1 | 1 |
| 3-5 | 2 | 5 | 14 |
| 4-6 | 2 | 5 | 8 |
| 5-6 | 3 | 6 | 15 |



(*i*) Draw the project network.

(*ii*) Find the expected duration and variance for each activity. What is the expected project length ?

(*c*) Consider the following data of the project :

| Activity | Predecessor (s) | Duration (weeks) | | |
|---|---|---|---|---|
| | | $t_o$ | $t_m$ | $t_p$ |
| *A* | – | 3 | 5 | 8 |
| *B* | – | 6 | 7 | 9 |
| *C* | – | 4 | 5 | 9 |
| *D* | *A* | 3 | 5 | 8 |
| *E* | *B* | 4 | 6 | 9 |
| *F* | *A* | 5 | 8 | 11 |
| *G* | *C*, *D* | 3 | 6 | 9 |
| *H* | *C*, *D*, *E* | 1 | 2 | 9 |

(*i*) Construct the project network.

(*ii*) Find critical path and expected completion time.

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