

# DD-762

## M. A./M. Sc. (Fourth Semester) EXAMINATION, 2020

MATHEMATICS

Paper First

(Functional Analysis—II)

*Time : Three Hours*

*Maximum Marks : 80*

**Note :** Attempt any *two* parts from each question. All questions carry equal marks.

1. (a) State and prove uniform boundedness theorem.  
(b) State and prove open mapping theorem.  
(c) Let  $T$  be a closed linear map of a Banach space  $X$  into a Banach space  $Y$ . Then  $T$  is continuous.
2. (a) State and prove Hahn-Banach theorem for real linear space.  
(b) A closed subspace of a reflexive Banach space is reflexive.  
(c) State and prove closed range theorem.
3. (a) Every inner product space is a normed space but converse need not be true.

P. T. O.

- (b) Give the definition of orthonormal set and let  $S = \{x_1, x_2, \dots\}$  be linearly independent sequence in an inner product space. Then there exists an orthonormal sequence  $T = \{y_1, y_2, \dots\}$  such that  $L(S) = L(T)$ .
- (c) Let  $\{e_i\}$  be a non-empty arbitrary orthonormal set in a Hilbert space  $H$ . Then the following conditions are equivalent :
- (i)  $\{e_i\}$  is complete
  - (ii)  $x \perp \{e_i\} \Rightarrow x = 0$
  - (iii)  $x \in H \Rightarrow x = \sum (x, e_i) e_i$
  - (iv)  $x \in H \Rightarrow \|x\|^2 = \sum_i |(x, e_i)|^2$
4. (a) A closed convex subset  $C$  of a Hilbert space  $H$  contains a unique vector of smallest  $n$  or  $m$ .
- (b) Let  $M$  be a proper closed linear subspace of a Hilbert space  $H$ . Then there exists a non-zero vector  $z_0$  in  $H$  s. t.  $z_0 \perp M$ .
- (c) State and prove projection theorem.
5. (a) Let  $T$  be an operator on  $H$ . Define the adjoint  $T^*$  of  $T$ . The mapping  $T \rightarrow T^*$  of  $B(H)$  into itself has the following properties : For  $T, T_1, T_2 \in B(H)$  and  $\alpha \in C$  :
- (i)  $I^* = I$ , where  $I$  is the identify operator
  - (ii)  $(T_1 + T_2)^* = T_1^* + T_2^*$
  - (iii)  $(\alpha T)^* = \alpha T^*$
  - (iv)  $(T_1 T_2)^* = T_2^* T_1^*$

- (b) If  $T_1$  and  $T_2$  are normal operators on a Hilbert space  $H$  with the property that either commutes with the adjoint of the other then  $T_1 + T_2$  and  $T_1T_2$  are normal.
- (c) Let  $T$  be a bounded linear operator on a Hilbert space  $H$ . Then :
- (i)  $T$  is normal  $\Leftrightarrow \|T^*x\| = \|Tx\| \quad \forall x \in H$
- (ii) If  $T$  is normal, then  $\|T^2\| = \|T\|^2$

[https://universitynews.in](https://universitynews.in/)