Roll No. ....

DD–762

M. A./M. Sc. (Fourth Semester) EXAMINATION, 2020

MATHEMATICS

Paper First

(Functional Analysis—II)

Time : Three Hours

Maximum Marks : 80

Note : Attempt any *two* parts from each question. All questions carry equal marks.

1. (a) State and prove uniform boundedness theorem.

   (b) State and prove open mapping theorem.

   (c) Let T be a closed linear map of a Banach space X into a Banach space Y. Then T is continuous.

2. (a) State and prove Hahn-Banach threorem for real linear space.

   (b) A closed subspace of a reflexive Banach space is reflexive.

   (c) State and prove closed range theorem.

3. (a) Every inner product space is a normed space but converse need not be true.

(b) Give the definition of orthonormal set and let $S = \{x_1, x_2 ....\}$ be linearly independent sequence in an inner product space. Then there exists an orthonormal sequence $T = \{y, y_2, ...\}$ such that L (S) = L (T).

(c) Let $\{e_i\}$ be a non-empty arbitrary orthonormal set in a Hilbert space H. Then the following conditions are equivalent :

(i) $\{e_i\}$ is complete

(ii) $x \perp \{e_i\} \Rightarrow x = 0$

(iii) $x \in H \Rightarrow x = \Sigma (x, e_i)\, e_i$

(iv) $x \in H \Rightarrow \| x \|^2 = \sum_i |(x, e_i)|^2$

4. (a) A closed convex subset C of a Hilbert space H contains a unique vector of smallest *n* or *m*.

(b) Let M be a proper closed linear subspace of a Hilbert space H. Then there exists a non-zero vector z_0 in H s. t. $z_0 \perp M$.

(c) State and prove projection theorem.

5. (a) Let T be an operator on H. Define the adjoint T^* of T. The mapping $T \to T^*$ of B (H) into itself has the following properties : For T, T_1, T_2 $\in \beta(H)$ and $\alpha \in C$:

(i) $I^* = I$, where I is the identify operator

(ii) $(T_1 + T_2)^* = T_1^* + T_2^*$

(iii) $(\alpha T)^* = \alpha T^*$

(iv) $(T_1 T_2)^* = T_2^* T_1^*$

(b) If T_1 and T_2 are normal operators on a Hilbert space H with the property that either commutes with the adjoint of the other then $T_1 + T_2$ and T_1T_2 are normal.

(c) Let T be a bounded linear operator on a Hilbert space H. Then :

(i) T is normal $\Leftrightarrow \|T^* x\| = \|Tx\| \;\; \forall\, x \in H$

(ii) If T is normal, then $\|T^2\| = \|T\|^2$