

Roll No.

DD-771

M. A./M. Sc. (Fourth Semester) EXAMINATION, 2020

MATHEMATICS

(Optional—B)

Paper Fifth

(Graph Theory)

Time : Three Hours

Maximum Marks : 80

Note : Attempt any *two* parts from each question. All questions carry equal marks.

Unit—I

1. (a) For any $s, t \geq 1$, prove that there is $R(s, t) < \infty$ such that any graph on $R(s, t)$ vertices contains either an independent set of size s or a clique of size t . In particular :

$$R(s, t) \leq \binom{s+t-2}{s}.$$

- (b) Prove that every graph on $\begin{bmatrix} k+l \\ k \end{bmatrix}$ vertices contains either a complete sub-group on $k+1$ vertices or an independent set of $l+1$ vertices.
- (c) Prove that for any $s \geq 2$,

$$R(s, s) \geq 2^{\frac{s}{2}}.$$

Unit—II

2. (a) Prove that every vertex of a composite connected graph lies on a 4-cycle.
- (b) Prove that the line-group and the point group of a graph G are isomorphic iff G has at most one isolated point and k_2 is not a component of G .
- (c) Prove that the group of the union of two graphs is the sum of their groups :

$$\Gamma(G \cup G_2) \equiv \Gamma(G_1) + \Gamma(G_2)$$

iff no component of G , is isomorphic with a component of G_2 .

Unit—III

3. (a) Prove that every 3-chromatic maximal planar graph is uniquely 3-colorable.
- (b) If B_1, B_2, \dots, B_r are the blocks of a graph G , then prove that :

$$\phi(G, x) = \frac{1}{x^{r-1}} \prod_{i=1}^{\infty} Q(B_i, x).$$

- (c) Prove that the coefficients of every chromatic polynomial alternate in sign.

Unit—IV

4. (a) Prove that each cycle $C_n, n \geq 3$ is chromatically unique.

(b) Prove that for any graph G :

$$\chi(G) \leq 1 + \max \delta(G'),$$

where the maximum is taken over all induced subgroups G' of G .

(c) Prove that for a connected graph G ,

$$T(G, 1, 1) = \tau(G),$$

the number of spanning trees of G .

Unit—V

5. (a) Prove that a graph G is n -line connected iff every pair of points are joined by at least n -line disjoint paths.

(b) State and prove Menger's theorem for digraph (vertex form).

(c) Prove that every acyclic graph without isolated has a unique basis consisting of its transmitters.