Roll No. ....

DD–763

M. A./M. Sc. (Fourth Semester)

EXAMINATION, 2020

MATHEMATICS

Paper Second

(Partial Differential Equations and Mechanics)

Time : Three Hours

Maximum Marks : 80

Note : All questions are compulsory. Attempt any *two* parts from each question. All questions carry equal marks.

Unit—I

1. (a) State and prove local existence theorem for first order non-linear PDE.

(b) Prove that the function x (.) solves the system of Euler-Lagrange's equations :

$$-\frac{d}{ds}\left(D_q L(\dot{x}(s), x(s))\right) + D_x L\left(\dot{x}(s), x(s)\right) = 0$$

$(0 \leq s \leq t)$

(c) State and prove Lax-Oleinik's formula.

P. T. O.

Unit—II

2. (a) Using the method of separation of variables solve the heat equation :

$$u_t - \Delta u = 0 \text{ in } U \times (0, \infty)$$

$$u = 0 \text{ on } \partial U \times [0, \infty)$$

$$u = g \text{ on } U \times \{t = 0\}$$

where $g : U \rightarrow R$ is given.

(b) Derive Barenblott's solution to the porous medium equation using similarity under scaling.

(c) Explain Hodograph transform.

Unit—III

3. (a) Suppose $k, l : R \rightarrow R$ are continuous functions, that l grows at most linearly and that k grows at least quadratically. Assume also there exists a unique point $y_0 \in R$ such that :

$$k(y_0) = \min_{y \in R} k(y)$$

Then prove that :

$$\lim_{\in \to 0} \frac{\int_{-\infty}^{\infty} l(y) e^{-\frac{k(y)}{\in}} dy}{\int_{-\infty}^{\infty} e^{-\frac{k(y)}{\in}} dy} = l(y_0).$$

(b) Discuss unperturbed PDE :

$$\text{div}(u\, b) = \delta_0 \text{ in } R^2$$

where δ_0 is the Dirac measure R^2 giving unit mass to the point O.

(c) Solve the wave equation using stationary phase method.

Unit—IV

4. (a) Derive mathematical expressions for Hamilton's principle.

(b) Derive Whittaker's equations.

(c) State and prove Lee Hwa-Chung's theorem.

Unit—V

5. (a) Derive Hamilton-Jacobi's equations.

(b) Prove that the Lagrange bracket is invariant under canonical transformation.

(c) Using Poisson brackets relation, show that the following transformation is canonical if $ad - bc = 1$:

$$Q = aq + bp$$
$$P = cq + dp$$