

DD-763

M. A./M. Sc. (Fourth Semester)

EXAMINATION, 2020

MATHEMATICS

Paper Second

(Partial Differential Equations and Mechanics)

Time : Three Hours

Maximum Marks : 80

Note : All questions are compulsory. Attempt any *two* parts from each question. All questions carry equal marks.

Unit—I

- (a) State and prove local existence theorem for first order non-linear PDE.
- (b) Prove that the function $x(\cdot)$ solves the system of Euler-Lagrange's equations :

$$-\frac{d}{ds} D_q L(\dot{x}(s), x(s)) + D_x L(\dot{x}(s), x(s)) = 0$$

$$(0 \leq s \leq t)$$

- (c) State and prove Lax-Oleinik's formula.

Unit—II

2. (a) Using the method of separation of variables solve the heat equation :

$$u_t - \Delta u = 0 \text{ in } U \times (0, \infty)$$
$$u = 0 \text{ on } \partial U \times [0, \infty)$$
$$u = g \text{ on } U \times \{t = 0\}$$

where $g : U \rightarrow \mathbb{R}$ is given.

- (b) Derive Barenblott's solution to the porous medium equation using similarity under scaling.
(c) Explain Hodograph transform.

Unit—III

3. (a) Suppose $k, l : \mathbb{R} \rightarrow \mathbb{R}$ are continuous functions, that l grows at most linearly and that k grows at least quadratically. Assume also there exists a unique point $y_0 \in \mathbb{R}$ such that :

$$k(y_0) = \min_{y \in \mathbb{R}} k(y)$$

Then prove that :

$$\lim_{\epsilon \rightarrow 0} \frac{\int_{-\infty}^{\infty} l(y) e^{-\frac{k(y)}{\epsilon}} dy}{\int_{-\infty}^{\infty} e^{-\frac{k(y)}{\epsilon}} dy} = l(y_0).$$

- (b) Discuss unperturbed PDE :

$$\operatorname{div}(u b) = \delta_0 \text{ in } \mathbb{R}^2$$

where δ_0 is the Dirac measure \mathbb{R}^2 giving unit mass to the point O.

- (c) Solve the wave equation using stationary phase method.

Unit—IV

4. (a) Derive mathematical expressions for Hamilton's principle.
(b) Derive Whittaker's equations.
(c) State and prove Lee Hwa-Chung's theorem.

Unit—V

5. (a) Derive Hamilton-Jacobi's equations.
(b) Prove that the Lagrange bracket is invariant under canonical transformation.
(c) Using Poisson brackets relation, show that the following transformation is canonical if $ad - bc = 1$:

$$Q = aq + bp$$

$$P = cq + dp$$