Roll No. ....

ED–2814

M. A./M. Sc. (Final) EXAMINATION, 2021

MATHEMATICS

(Optional)

Paper Fifth (*iii*)

(Fuzzy Sets and Their Application)

Time : Three Hours

Maximum Marks : 100

Note : Attempt any *two* parts from each question. All questions carry equal marks.

1. (a) If :

$$A = \frac{1}{1.0} + \frac{2}{0.8} + \frac{3}{0.5} + \frac{4}{0.1}$$

is defined on $X = \{1, 2, 3, 4\}$. Find all its α-cuts sets. Also prove that A can be expressed in terms of the family composed of all of its α-cuts.

(b) Define t-norm with example and prove that the standard fuzzy intersection is the only idempotent t-norm.

P. T. O.

(c) If A is a fuzzy set defined on X, then prove that $\bigcup_{\alpha \in [0,1]} {}_{\alpha+}A = A_1$ where ${}_{\alpha+}A$ is a special set and ${}_{\alpha+}A(x) = \alpha . {}^{\alpha+}A(x)$.

2. (a) If R is fuzzy relation on X^2. Then prove that fuzzy relation :

$$R_{T(i)} = \bigcup_{n=1}^{\infty} R^{(n)}$$

is the smallest *i*-transitive closure of R.

(b) Define symmetric fuzzy relation with example. If R is symmetric fuzzy relation, then prove that each power of R is symmetric.

(c) What do you mean by sagittal diagram. Draw the sagittal diagram for the following fuzzy relation R :

$$R = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{array} \begin{array}{c} \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \begin{bmatrix} .8 & 0 & .3 & .4 \\ .3 & 1 & .8 & 0 \\ .7 & 0 & 1 & 1 \\ .4 & .5 & 0 & 0 \\ 0 & 1 & .5 & .8 \end{bmatrix} \end{array}$$

3. (a) Solve the following fuzzy relation equation using max-min operations :

$$P \; O \begin{bmatrix} .9 & .6 & 1 \\ .8 & .8 & .5 \\ .6 & .4 & .6 \end{bmatrix} = [.6 \;\; .6 \;\; .5]$$

(b) If m_1 and m_2 are basic probability assignments on $X = \; a,b,c,d$ which are obtained from two independent sources be defined as follows :

$m_1 \;\; a,b \;\; = .2$ $\qquad$ $m_1 \;\; b,d \;\; = .5$

$m_1 \;\; a,c \;\; = .3$ $\qquad$ $m_2 \;\; a,d \;\; = .2$

$m_2 \;\; b,c \;\; = .5$ $\qquad$ $m_2 \;\; a,b,c \;\; = .3$

Calculate the combined basic probability assignment $m_{1.2}$ by using the Dempster rule of combination.

(c) Prove that :

(i) bel (A) + bel $(\bar{A}) \leq 1$

(ii) pl (A) + pl $(\bar{A}) \geq 1$

4. (a) What do you mean by interpolation method ? If A_1, A_2, B_1, B_2 are fuzzy sets such that :

$$A_1 = \frac{1}{x_1} + \frac{.9}{x_2} + \frac{.1}{x_3}$$

$$A_2 = \frac{.9}{x_1} + \frac{1}{x_2} + \frac{.2}{x_3}$$

$$B_1 = \frac{1}{y_1} + \frac{.2}{y_2}$$

$$B_2 = \frac{.2}{y_1} + \frac{.9}{y_2}$$

X is A is fact where $A = \frac{.8}{x_1} + \frac{.9}{x_2} + \frac{.1}{x_3}$

Then calculate the conclusion B by method of interpolation.

(b) Write short notes on the following :

(i) unconditional and unqualified proposition

P. T. O.

(ii) relative quantifier

(iii) multivalued logic

(iv) linguistic hedges

(c) If $X = \{1, 2, 3, 4\}$ and $Y = \{1, 2, 3, 4, 5, 6\}$ are two universe of discourse and

$$A = \frac{0.6}{2} + \frac{1}{3} + \frac{0.2}{4}$$

$$B = \frac{0.4}{2} + \frac{1}{3} + \frac{0.8}{4} + \frac{0.3}{5}$$

Apply the fuzzy modus ponens rule to find the relation. If *x* is A, then *y* is B.

5. (a) If each individual of four decisions makes has a total preference ordering $P_i \ (i \in N)$ on a set of alternatives $X = \{a, b, c, d\}$ as :

$P_1 = \{a, b, d, c\}$

$P_2 = \{a, c, b, d\}$

$P_3 = \{b, a, c, d\}$

$P_4 = \{a, d, b, c\}$

Find the fuzzy preference relation. Also find the α-cuts of the fuzzy relation.

(b) Solve the following fuzzy linear programming problem :

Maximize :

$$z = 5x_1 + 4x_2$$

$$(4, 2, 1)\, x_1 + (5, 3, 1)\, x_2 \le (24, 5, 8)$$

$$(4, 1, 2)\, x_1 + (1, .5, 1)\, x_2 \le (12, 6, 3)$$

$$x_1, x_2 \ge 0$$

(c) Discuss the methods of defuzzification.