ED-2806

M.A./M.Sc. (Final) Examination, 2021

MATHEMATICS

Compulsory

Paper - I

Integration Theory and
Functional Analysis

Time : Three Hours] [*Maximum Marks* : 100

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) State and prove Hahn decomposition theorem.

(*b*) State and prove Riesz representation theorem.

(*c*) State and prove Fubini's theorem.

Unit-II

2. (*a*) Let μ be a finite measure defined on a σ algebra $\mathscr{A}$ which contains all the baire sets for locally compact space X. If μ is inner regular, then it is regular.

(*b*) Let μ be a measure defined on a σ algebra $\mathscr{A}$ containing the baire sets. Assume that eigher μ is quasi regular or μ is inner regular. Then for each $E \subset \mathscr{A}$ with $\mu(E) < \infty$ there is a baire set B with

$$\mu(E \,\Delta\, B) = 0$$

(*c*) The intersection of a sequence of inner regular sets of finite measure is inner regular. Also the intersection of a decreasing sequence of outer regular sets of finite measure is outer regular.

Unit-III

3. (*a*) Let M be closed linear subspace of a normed linear space N. If the norm of the coset $x + M$ in the quotient space N/M is defiend by

$$\| x + M \| = \inf \{\| x + m \| : m \in M\}$$

Then N/M is normed linear space. Further if N is a Banach space, then so is N/M.

(*b*) Show that all norms are equivalent on a finite dimensional space.

(*c*) Let X and Y be normed linear spaces and T a linear transformation on X into Y. Then T is continuous either at every point of X or at no pont of X. It is continuous on X if and only if there is a constant M such that

$$\| T_x \| < M \| x \| \text{ for every } x \text{ in } X.$$

Unit-IV

4. (*a*) State and prove Hahn Banach theorem for real linear space.

(*b*) Let $\{T_n\}$ be a sequence of compact linear operators from a normed space X into a Banach space Y and T be a bounded linear operator, $T : X \to Y$ such that $\| T_n - T \| \to 0$ as $n \to \infty$, then the limit operator T is compact.

(*c*) State and prove closed range theorem.

Unit-V

5. (*a*) State and prove projection theorem.

(*b*) If H is a Hilbert space, then H is reflexive.

(*c*) Any arbitrary operator T on a Hilbert space H can be uniquely expressed as $T = T_1 + iT_2$, where T_1 and T_2 are self adjoint operator.