**No. of Printed Pages : 4** **Roll No……**

# ED-2805

## M.A./M.Sc. (Previous)
## EXAMINATION, 2021

**MATHEMATICS**

**Paper Fifth**

**(Advance Discrete Mathematics)**

**Time : Three hours** **Maximum Marks : 100**

**Note :** *Attempt any two parts from each question. All questions carry equal marks.*

**Unit–1**

**1.** (a) State and prove Basic Homomorphism Theorem.

(b) Define the following :

(i) Tautology,

(ii) Semigroup,

(iii) Congruence Relation,

(iv) Submonoid,

(v) Direct Product.





(c) Define Monoid and show that for any commulative monoid (M, *), the set of idempotent elements of M forms a sub-monoid.

**Unit–2**

**2.** (a) Define Distributive lattice and show that the direct product of any two distributive lattice is a distributive lattice.

(b) Discuss Karnaugh map with one, two, three and four variables.

(c) Write the following Boolean expressions in an equivalent sum of products canonical form in three variables $x_1, x_2$ and $x_3$ :

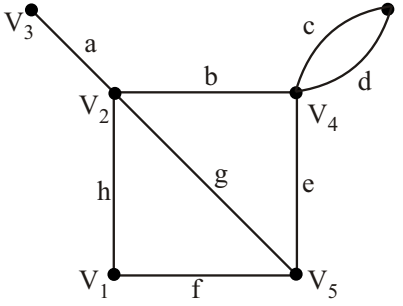
(i) $x_1 * x_2$

(ii) $x_1 \quad x_2$

(iii) $(x_1 \quad x_2) * x_3$

**Unit–3**

**3.** (a) Define circuit matrix and find the circuit matrix in given graph :

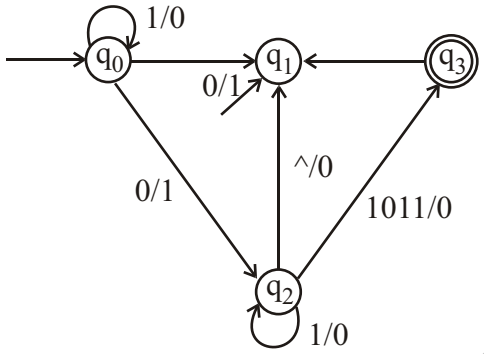



(b) Discuss about Warshall's Algorithm.

(c) Define graph and show that a simple graph (i.e. a graph without parallel edges or self loops) with $n$ vertices and $k$ components can have at most $\frac{(n \quad k)(n \quad k \quad 1)}{2}$ edges.

**Unit–4**

**4.** (a) Define transition system and consider the transition system in the following fig. Determine the initial states, final states and the acceptability of 101011, 111010.




(b) Define non-deterministic finite automata and a deterministic acceptor equivalent M = $(\{q_0, q_1, q_2\}, \{a, b\}, \quad q_0, \{q_2\})$ is given following table :

**State Table**

| State/ | a | b |
|---|---|---|
| $q_0$ | $q_0, q_1$ | $q_2$ |
| $q_1$ | $q_0$ | $q_1$ |
| $q_2$ | | $q_0, q_1$ |



(c) Let $S$ be any state in a finite state machine and $x$ and $y$ be any words, then prove :

$(S, xy) \quad ( \ (S, x), y)$

and $(S, xy) \quad ( \ (S, x), y)$

**Unit–4**

**5.** (a) Define the following :

(i) Grammar

(ii) Regular set

(iii) Context sensitive grammar

(iv) Notions of Syntax Analysis

(v) Conversion of Inflix Expressions to Polish Notation.

(b) Define Polish notation. Convert the expression $a \quad b \quad \frac{c.d}{g \quad f}$ into Polish notation.

(c) Define Language and let language $L(G_S) \quad \{a^n b^n c^n \mid n \quad 1\}$ is generated by the following grammar $G_S \quad \{S, B, C\}, \{a, b, c\}, S,$ where consists of the production.

$S \quad aSBC, S \quad aBC, CB \quad BC, aB \quad ab,$
$bB \quad bb, bC \quad bc, cC \quad cc.$

Then find the derivation for the strings $abc$, $a^2b^2c^2$ and $a^3b^3c^3$.