ED-2804

M.A./M.Sc. (Previous) Examination, 2021

MATHEMATICS

Paper - IV

Complex Analysis

Time : Three Hours] [*Maximum Marks* : 100

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) State and prove Laurent's theorem.

(*b*) Prove that equation $2z^5 + 8z - 1 = 0$ has one real root in $|z| < 1$ and four roots lie between the circles $|z| = 1$, and $|z| = 2$.

(*c*) Let $f(z)$ be analytic in a neighbourhood of z_0 and $f'(z_0) \neq 0$, then the relation $w = f(z)$ defines z as analytic function $f^{-1}(w)$ in some neighbourhood of the point $w_0 = f(z_0)$.

Unit-II

2. (*a*) If $0 < a < 1$, then use the method of contour integration to evaluate

$$\int_0^\infty \frac{x^{a-1}}{1-x}\,dx$$

(*b*) Find the image of the circle $|z-2|=2$ under the Mobius transformation $w = \frac{z}{z+1}$.

(*c*) State and prove Hurwitz's theorem.

Unit-III

3. (*a*) For Re $z > 1$, prove that

$$\zeta(z)\cdot\Gamma z = \int_0^\infty \left(e^t - 1\right)^{-1} t^{z-1}\,dt$$

(*b*) Let $f(z)$ be an analytic function in a domain D containing a segment of the x-axis and is symmetric to the x-axis. The show that

$$\overline{f(z)} = -f(\overline{z}),\ z \in D$$

if and only if $f(x)$ is purely imaginary for each point on the segment of x-axis.

(*c*) Let $D = \{ z : |z| < 1\}$ be the unit disk and suppose that $f : \partial D \to \mathbb{R}$ is a continous function, then there *iD* a continuous function $u : \overline{D} \to \mathbb{R}$ such that $u(z)$ is harmonic in D.

Unit-IV

4. (*a*) State and prove Poisson-Jensen formula.

(*b*) What do you mean by exponents of convergence ? If f is an entire function of finite order λ then f has finite genus $\mu \leq \lambda$.

(*c*) The order of a canonical product is equal to the exponent of convergence of its zeros.

Unit-V

5. (*a*) Let f be an analytic function in a region containing $\overline{B}(0; R)$, then $f(B(0; R)$ contains a disk of radious $\frac{1}{72} \cdot R|f'(0)|$.

(*b*) State and prove Great Picard theorem.

(*c*) Define univalent function and show that a univalent function that maps $|z| < \infty$ onto $|w| < \infty$ must be linear.