ED-2802

M.A./M.Sc. (Previous) Examination, 2021

MATHEMATICS

Paper - II

Real Analysis

Time : Three Hours] [*Maximum Marks* : 100

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) (*i*) Define unit step function.

(*ii*) Suppose $C_n \geq 0$ for $n = 1, 2, 3, ... \sum C_n$ converges, $\{s_n\}$ is a sequence of distinct points in (a, b) and

$$\alpha(a) = \sum_{n=1}^{\infty} C_n I(x - s_n).$$

Let f be continuous on $[a, b]$; then prove that $\int_a^b f d_\alpha = \sum_{n=1}^{\infty} C_n f(s_n)$.

(*b*) Let $I = [0,\ 1]$ and let $f, \alpha : I \to R$ be function such that $f(x) = \alpha(x) = x^2$. Then find the value of $\int_0^1 x^2 dx$.

(*c*) Let $Y : [a,b] \to R^k$ be a curve. If $C \in (a,b)$, then prove that $\Lambda_Y(a,b) = \Lambda_Y(a,c) + \Lambda_Y(c,b)$.

Unit-II

2. (*a*) State and prove the Riemann's theorem on rearrangement of series.

(*b*) State and prove the Abel's test for uniform convergence.

(*c*) State and prove the converse of Abel's theorem.

Unit-III

3. (*a*) State and prove the Chain Rule.

(*b*) Show that the volume of the greatest rectangular parallelepiped inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ is $\frac{\delta abc}{3\sqrt{3}}$.

(*c*) Let ψ be a k-chain of class $\mathcal{C}$ in an open set $V \subset R^m$ and let ω be a $(k-1)$-form of class $\mathcal{C}$ in V. The prove that $\int_{\psi} d\omega = \int_{\partial\psi} \omega$.

Unit-IV

4. (*a*) Prove that the outer measure of an interval is its length.

(*b*) State and prove the lebesgue's dominated convergence theorem.

(*c*) State and prove the fundamental theorem of integral calculus.

Unit-V

5. (*a*) Prove that the set function μ^* is an outer measure.

(*b*) State and prove the Riesz-Fischer theorem.

(*c*) State and prove the Riesz theorem.

https://universitynews.in